

Epistemic Uncertainty in the Prediction of Engineering Systems

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Outline of Presentation

- Programmatic background
- Epistemic uncertainty project
- Epistemic uncertainty in computational modeling
- Example problem
- Project activities
- Status and future work

Programmatic Background



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- **ASCI Verification and Validation Program Element:**
 - **Program manager: Marty Pilch, Org. 9133**
 - **Technical lead: Tim Trucano, Org. 9211**
- **Major tasks in the V&V program:**
 - **Planning and assessment of verification, validation, and software quality assurance practices**
 - **Validation metrics**
 - **Validation simulations for stockpile-to-target-simulations (normal, abnormal, and hostile environments)**
 - **Uncertainty quantification**

ASCI Epistemic Uncertainty Projects



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- **Non-distributional probabilistic approach:**
 - John Red-Horse, PI
 - Relaxation of full specification of probability distributions
- **Epistemic uncertainty project:**
 - Bill Oberkampf, PI
 - Generalization of traditional probability theory
- **Probabilistic robustness analysis:**
 - Steve Wojtkiewicz, PI
 - Propagation of interval and random variable uncertainty

Epistemic Uncertainty Project



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- Began in Q3 of FY00
- Team Members:
 - Bill Oberkampf (fluid dynamics and heat transfer)
 - Jon Helton (traditional probabilistic risk assessment)
 - Cliff Joslyn (generalized information theory)
 - Scott Ferson (non-traditional probabilistic risk assessment)
 - Steve Wojtkiewicz (traditional probabilistic dynamics)
 - Kari Sentz (generalized information theory)
- Project goal:
 - Evaluate feasibility of generalizations of probability theory for uncertainty estimation in large-scale simulations.

Epistemic Uncertainty Project Approach



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- **Methods must be directly compatible with traditional probabilistic approaches.**
- **Emphasis is on improving uncertainty estimation when data is scarce.**
- **Methods must be usable with existing large-scale computational codes, i.e., “black box” codes**
- **Improvement of non-traditional methods over traditional methods will be judged based on comparison of:**
 - **Ability to construct uncertainty representations of input data**
 - **Fidelity of mapping input uncertainty to output uncertainty**
 - **Required number of computational model function evaluations**
 - **Ability to conduct sensitivity analyses**

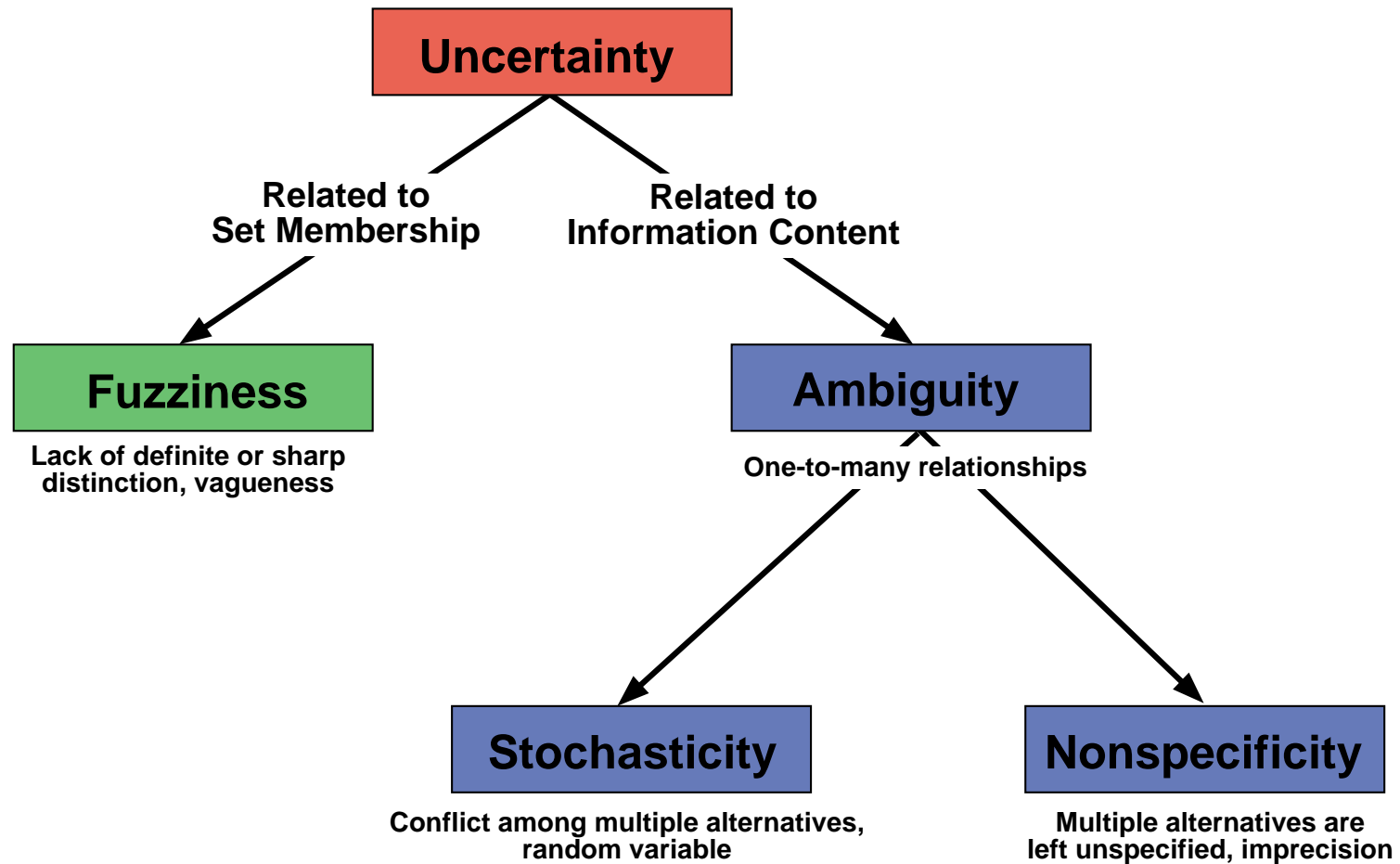
Epistemic Uncertainty in Computational Modeling



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- **Parametric** uncertainty:
 - Uncertainty characterized by a probability density function with interval valued parameters
 - Expert opinion input characterized by interval values and varying levels of belief over an interval
- **Mathematical modeling** uncertainty:
 - Uncertainty in the specification of an event tree or fault tree
 - Estimate uncertainty using alternate plausible mathematical models
- **Scenario abstraction** uncertainty:
 - Uncertainty in the identification of event or fault sequences
 - Uncertainty in the likelihood of the event or fault sequences

Generalized Types of Uncertainty



Evidence Theory as a Generalization of Probability Theory



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- **Non-additive measures of evidence theory:**

- **Belief measure:**

$$Bel(A) = \sum_{B \subseteq A} m(B)$$

- **Plausibility measure**

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B)$$

- **Belief and plausibility measures can be thought of as lower and upper probabilities, respectively:**

$$Bel(A) \leq Pl(A)$$

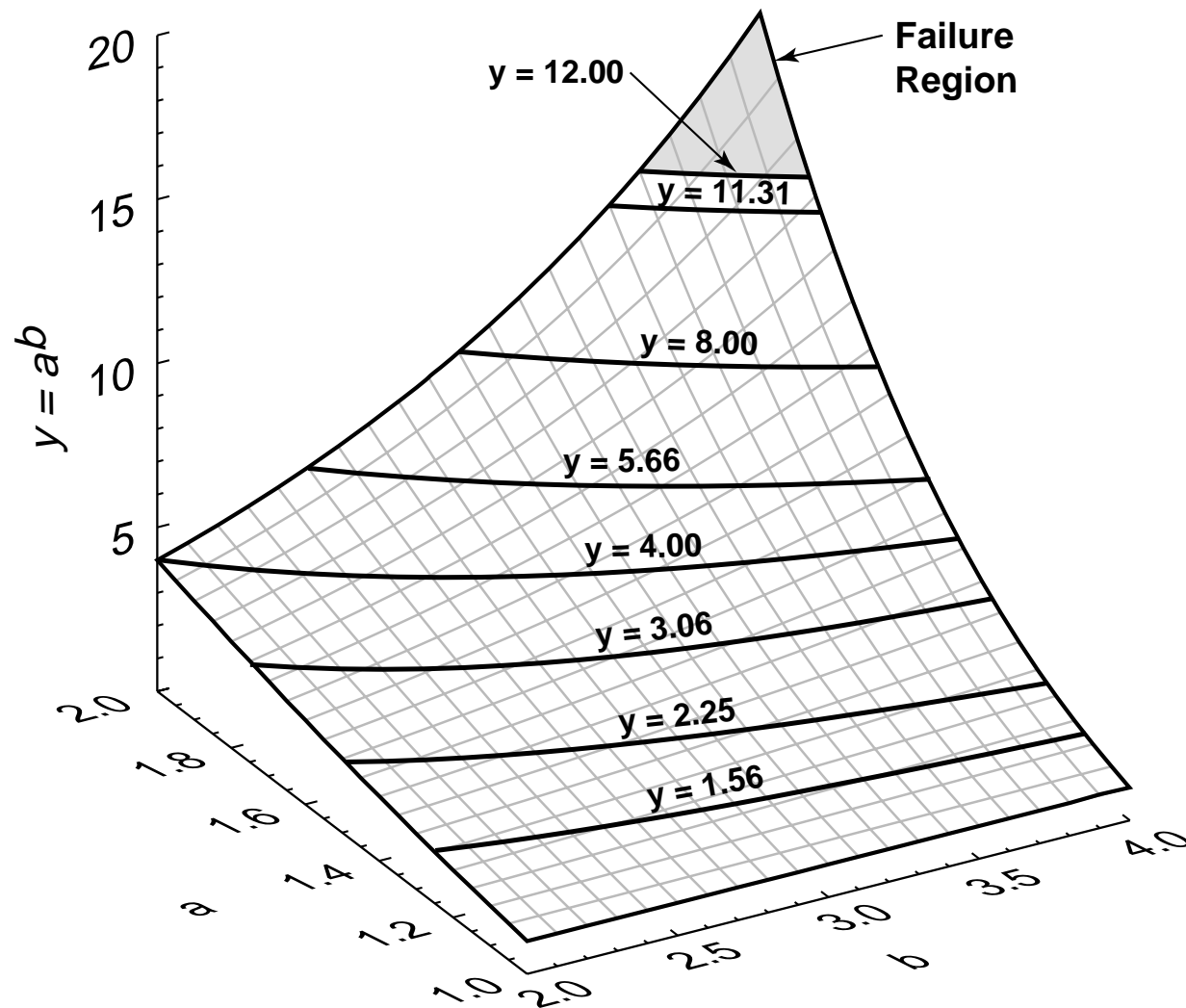
Example Problem

- Let the mathematical model for a physical process be given by

$$y = a^b$$

- Assume the only information concerning a and b is given by the nested sets:
 - $A_1 = [1, 1.25]$, $A_2 = [1, 1.5]$, $A_3 = [1, 1.75]$, and $A_4 = [1, 2]$
 - $B_1 = [2.5, 3]$, $B_2 = [2.5, 3.5]$, $B_3 = [2, 3.5]$, and $B_4 = [2, 4]$
- Assume the system fails when $y > 12$
- Goal of the analysis:
 - Given the data for a and b , what can be said about the occurrence of $y > 12$?

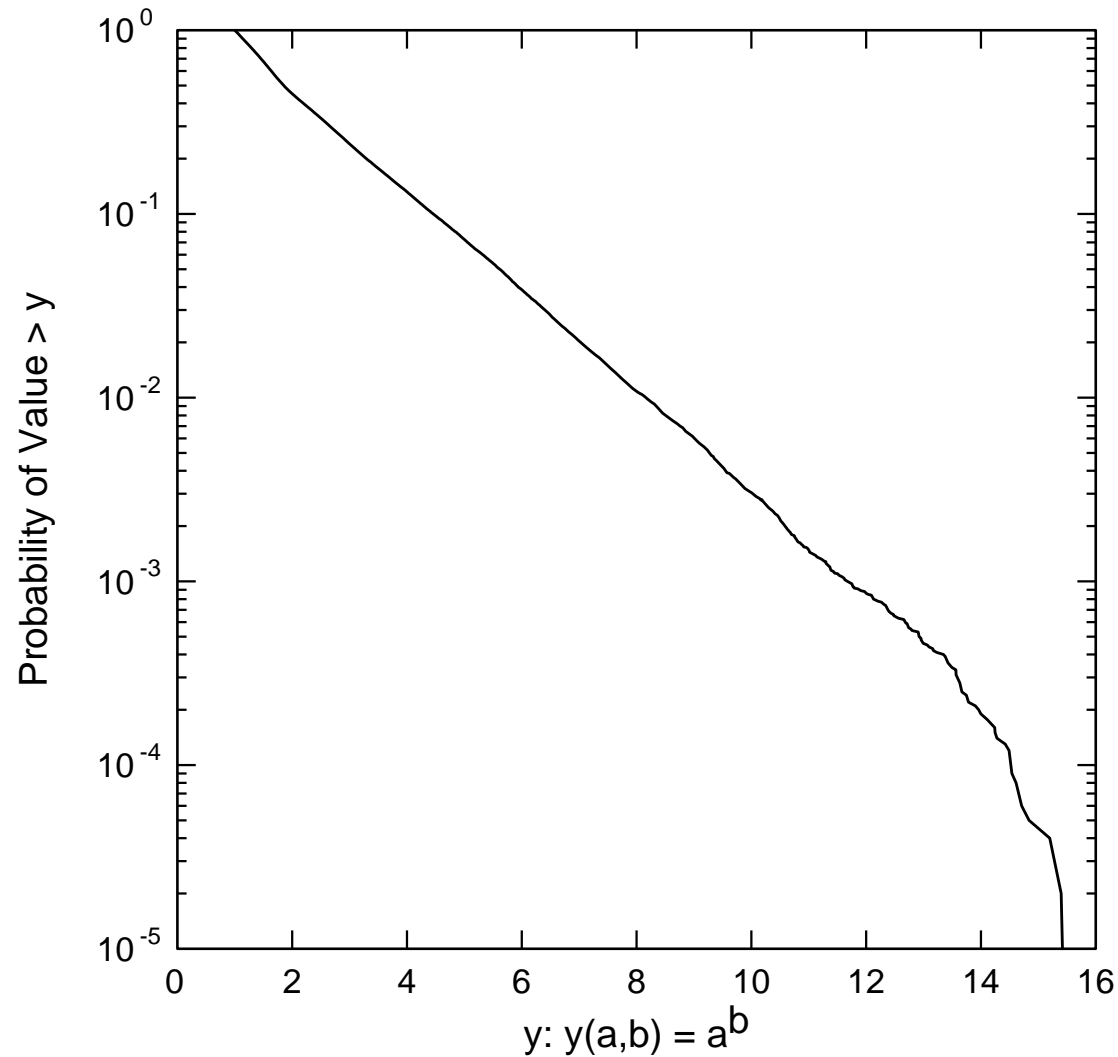
Character of the System Response



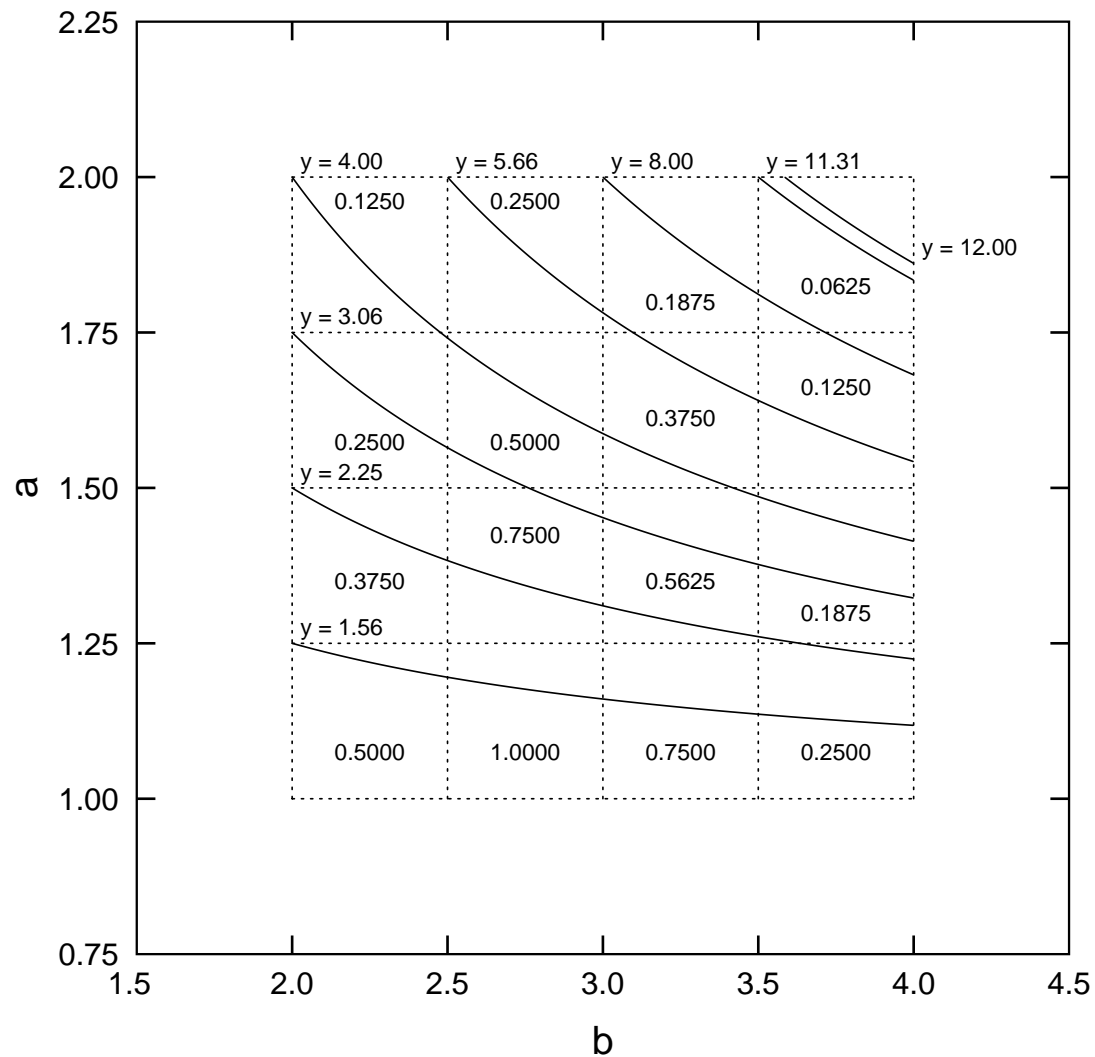
Solution from Traditional Probability Theory: Complementary CDF



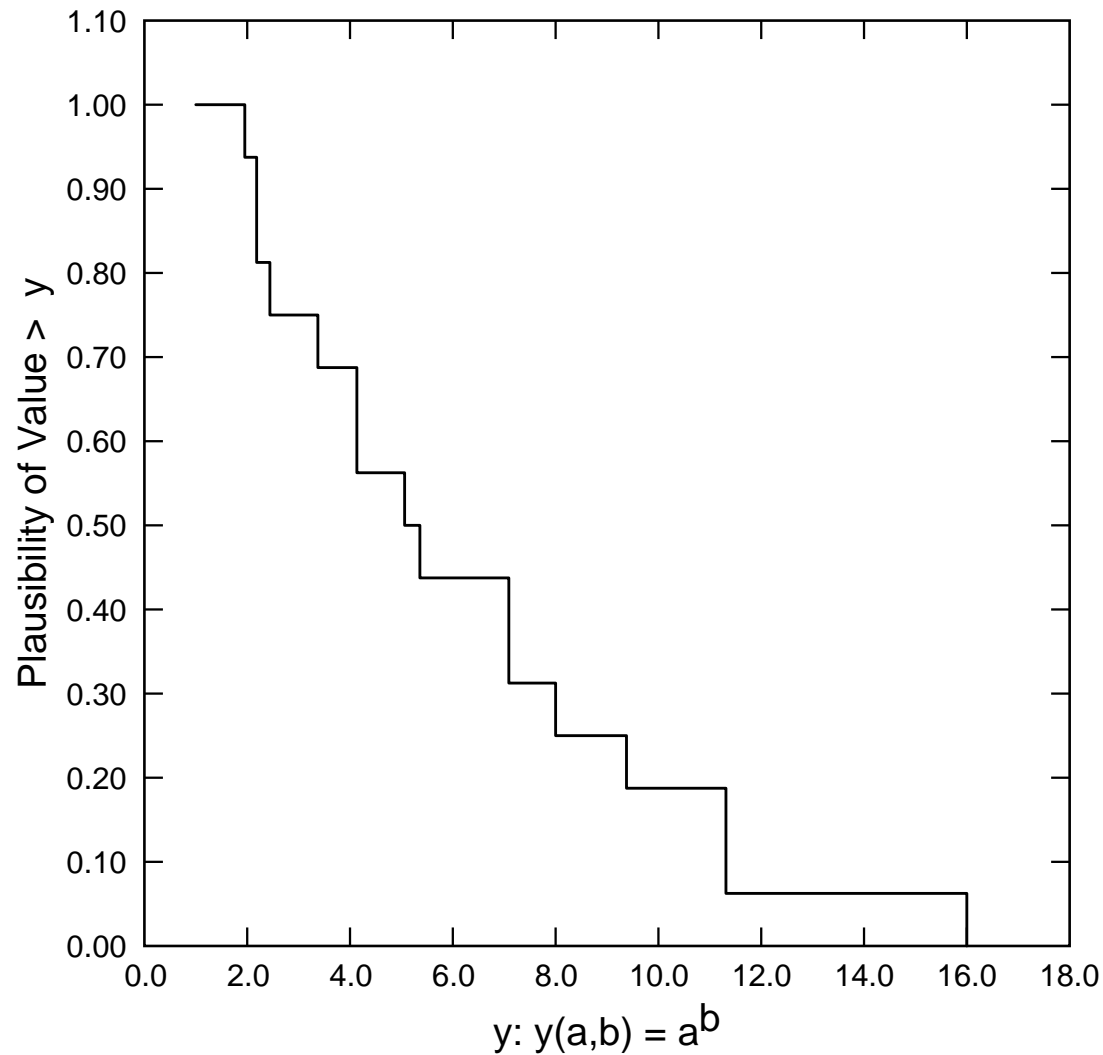
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Plausibility Values for the Product Space AXB



Solution with Evidence Theory: Complementary CPF



Project Activities



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- **Constructed a set of “Challenge Problems”:**
 - **Epistemic uncertainty exists only in input parameters for two simple computational models**
 - **Varying levels of epistemic uncertainty, from only interval values to specified PDFs with interval valued parameters**
 - **Two simple systems, algebraic system and an initial value problem given by a linear ordinary differential equation**
 - **Solutions from different approaches will be compared**
- **Constructed a web site describing the project and the challenge problems:**
 - **www.sandia.gov/epistemic/**

Workshop on Epistemic Uncertainty



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- Workshop will be held on August 6-7, 2002 in Albuquerque.
- Will have invited papers and submitted papers presenting solutions to one or more of the Challenge Problems.
- Papers can also pursue related topics in epistemic uncertainty
- Invited speakers cover a wide range of approaches:
 - Frequentist
 - Subjectivist
 - Information theory
 - Risk analysts
- Invited and submitted papers will be reviewed for publication in a special issue of *Reliability Engineering and System Safety*.

Status and Future Work



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- **Present work:**
 - **Methods for construction of uncertainty representations from a variety of sources with emphasis on scarce data**
 - **Survey of methods for aggregation of data with varying degrees of conflict**
 - **Proofs of convergence for Belief and Plausibility mapping of input to output**
 - **Improved understanding of factors affecting rate of convergence of Belief and Plausibility of input to output**
- **Future work:**
 - **Improved understanding of the relationship of probability-bounds to evidence theory**
 - **Development of convergence acceleration techniques for Belief and Plausibility**